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Brownian motion using video capture

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Abstract

Although other researchers had previously observed the random motion of pollen grains suspended in water through a microscope, Robert Brown's name is associated with this behaviour based on observations he made in 1828. It was not until Einstein's work in the early 1900s however, that the origin of this irregular motion was established to be the result of collisions with molecules which were so small as to be invisible in a light microscope (Einstein A 1965 *Investigations on the Theory of the Brownian Movement* ed R Furth (New York: Dover) (transl. Cowper A D) (5 papers)). Jean Perrin in 1908 (Perrin J 1923 *Atoms* (New York: Van Nostrand-Reinhold) (transl. Hammick D)) was able, through a series of painstaking experiments, to establish the validity of Einstein's equation. We describe here the details of a junior level undergraduate physics laboratory experiment where students used a microscope, a video camera and video capture software to verify Einstein's famous calculation of 1905.

1. Theory

In the case of a simple random walk problem in one dimension the square root of the average squared distance from the origin is proportional to the number of steps, N, which we show here as a simple example [3].

The displacement after one step is

$$x_1 = \pm 1.$$

Squared this is +1. After two steps the displacement is

$$x_2 = x_1 \pm 1.$$

Squaring this gives

$$x_2^2 = x_1^2 \pm 2x_1 + 1.$$

Now notice that $\langle x_1 \rangle_{ave} = 0$ so that taking the average of x_2^2 gives

$$\langle x_2^2 \rangle_{ave} = \langle x_1^2 \rangle_{ave} + 1 = 2.$$

From this logic we conclude that after N steps the average of the square of the location will be

$$\langle x_N^2 \rangle_{ave} = N.$$

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Einstein's 1905 paper used kinetic theory to show a similar result in three dimensions for a spherical particle being bombarded by the (smaller) molecules of a liquid in which the particle was immersed. In this case the number of steps is replaced by the number of collisions of the particle which is directly proportional to time. The size of the particle, the viscosity of the liquid in which the particle is immersed and the temperature are also factors. Einstein's equation for the average location of a particle making a random walk in a liquid is

$$\langle r^2 \rangle_{ave} = (RT/3^1 a\eta N_\mathrm{A})t$$

where R is the ideal gas constant, a is the radius of the particle, η is the viscosity of the liquid, N_A is Avogadro's number and t is the elapsed time. The displacement, r, is the magnitude of a three-dimensional location vector and appears in the equation as the average of the square of displacement. In his paper Einstein suggested that a measurement of the average position of a microscopic particle versus time could be used to make the first direct determination of Avogadro's number.

In 1908, Jean Perrin made a series of measurements verifying Einstein's equation and finding the first experimental determination of Avogadro's number. Using a process that took several months he first prepared tiny latex spheres of approximately equal radius. By following the motion of these spheres viewed under a microscope he was able to verify the relationship between time and average squared displacement for several different sized particles. In another set of measurements on the number density of particles suspended in a liquid Perrin determined Avogadro's number. He received the Nobel prize in physics in 1926 for this work.

2. Experimental set-up

We were able to avoid a lengthy particle preparation step by ordering a solution of plastic spheres (diameter 913 nm) from a scientific supply company (Sergent Welch [4]). One drop of sphere solution diluted with 20 ml of water provided an appropriate density of spheres for viewing.

A standard Olympus microscope at 400× power with a JVC TK1270 video camera attached through the RGB connection to a Sony SVO-2000 video recorder was used to collect images of the Brownian motion of the plastic spheres. The camera output was also fed to a TV monitor simultaneously to observe the image being recorded. After several trial runs, approximately 2 h of continuous video tape was taken to be analysed [5]. We found a vibration reduction table to be helpful in obtaining clear pictures over long data collection times.

The video tape was played back while connected to a Power Macintosh G3 with a graphics accelerator. The playback was captured at the recorded rate of 30 frames per second using Avid Cinema [6] for Macintosh. The final step was to determine the location of the particle at set time intervals using the program Videograph [7].

Videograph was first calibrated using a printed letter on a microscope slide. The height of the top of the letter 'e' was determined in the microscope using the graduated scale provided in the objective lens of the microscope (gradations on the scale are multiplied by a factor depending on the power used). The same letter was videotaped and used to calibrate the Videograph program. Any error in the calibration is squared in plots of average displacement squared so the results are particularly sensitive to the calibration procedure. We verified the calibration by measuring the distance across several of the plastic spheres (of known radius) which had stuck together and determined an error of only 1.4%.

A time interval of 2 s (every 60 video frames) gave an easily observed motion of the latex spheres. Longer time intervals were problematic in that the particle tended to wander off screen before enough points were measured. For shorter sampling intervals the motion of the particle was not far enough to be easily determined with Videograph. A total of 20 particles were observed and 30 locations were determined for each particle. Only particles which stayed in the focal plane of the microscope were used.

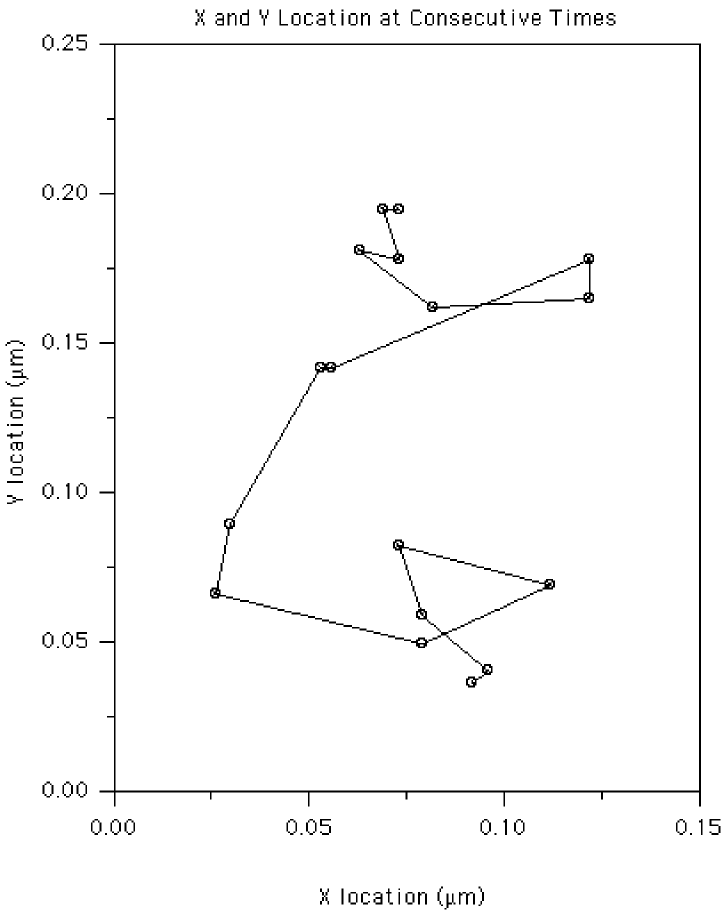

Figure 1. Random walk of one sphere at 0.03 s time intervals.

3. Results and suggestions for further projects

As can be seen in figure 1 the motion of a single particle appears to be quite random. The displacement of each particle from its initial location at each time step was squared and an average displacement was found for each of the 20 particles. The averages of the square of the position were plotted versus time (figure 2), a total of 600 data points. As can be seen from the graph, the data clearly show a linear proportionality between time and average squared position, as predicted by Einstein. The slope of the graph can be used to calculate a value of Avogadro's number, N_A (although Perrin used other measurements to find N_A). The data shown in figure 2 give a value for Avogadro's number of 4.2×10^{23} atoms mol^{-1}.

There are many other measurements related to Brownian motion which can easily be made using video equipment attached to a microscope. Perrin verified Einstein's equation by showing that the mean squared displacement was proportional to time for particles of different radii and liquids of different viscosity. This would be a straightforward extension of the present work. The equation also indicates a temperature dependence, which we did not explore. Perrin's actual determination of Avogadro's number was based on the law of atmospheres by finding the number density of his latex grains in a liquid as a function of height. The number of particles suspended at a particular height is determined by the interplay

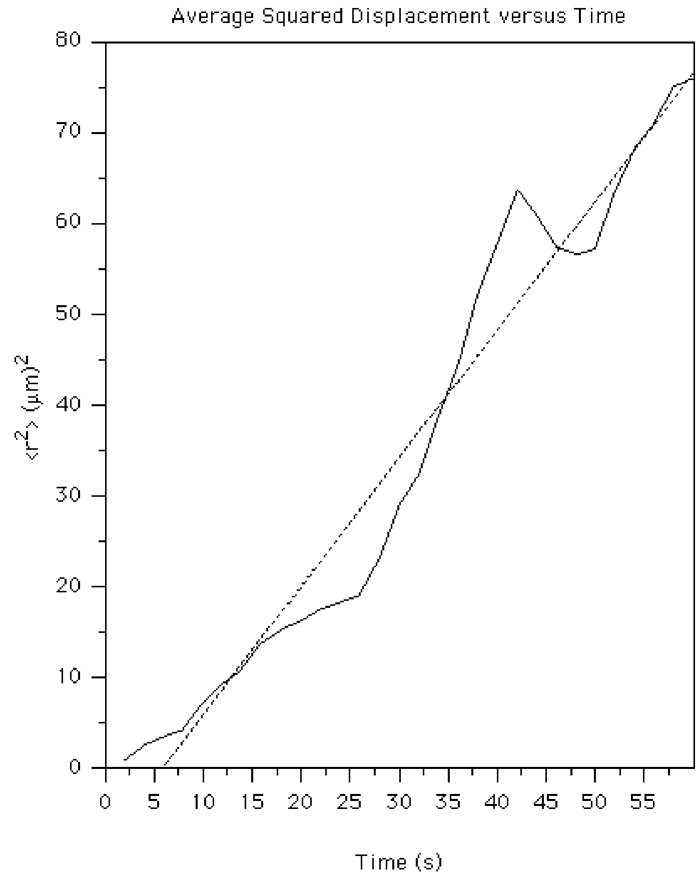

Figure 2. Plot of average square displacement versus time for 20 particles each at 30 time intervals of 2 s each.

between Brownian motion (which disperses these particles) and gravity (which tries to cause the particles to settle to the bottom of the container). Perrin also verified another of Einstein's equations which predicts the amount of rotation caused by molecular collisions. Diffusion is another phenomena which could be investigated with the same experimental apparatus. A discussion of experiments analysing variations to Einstein's original equation due to surface effects can be found in [8]. Theoretical considerations of Brownian motion and other kinds of random walk behaviour as it relates to chaos are found in [9].

Einstein's famous 1905 calculation firmly established the connection between macroscopic and microscopic physics and, as such, forms one of the pillars of the modern field of statistical mechanics. Our version of Perrin's verification of Einstein's prediction is relatively easy to set up, putting it within the reach of the typical undergraduate or high school laboratory experience.

Acknowledgment

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