

Additional Nonperturbative Contributions to Pion-Pion Scattering.

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Summary. - A cross-section for pion-pion scattering is calculated which includes contributions made by exact solutions of nonlinear equations. Results of this calculation are compared with cross-sections extrapolated from experimental data.

In theories of neutral, spin-zero mesons, it is well known that for self-coupling terms in the Lagrangian with powers of the field strength greater than four, the perturbation series diverges. This divergence cannot be removed by a simple cut-off procedure. A method of renormalization which avoids these problems for any power of the field strength has been devised by BURT⁽¹⁾ using exact nonperturbative solutions to the field equations. In this note, a calculation of meson-meson scattering amplitudes done by BURT⁽²⁾ using this approach is extended to higher orders by numerical methods and a scattering cross-section is found. Comparison is made to available experimental cross-sections.

The field equation describing neutral, spin-zero mesons is the nonlinear Kline-Gordon equation

$$(1) \quad \partial_\nu \partial^\nu \varphi + \varphi m^2 + \lambda \varphi^3 = 0.$$

This equation has been shown⁽³⁾ to have exact nonperturbative solutions of the form

$$(2) \quad \varphi_k^{(\pm)} = U_k^{(\pm)}(x) \left[1 - \frac{U_k^{(\pm)}(x)^2}{8m^2} \right]^{-1} = \sum_n \left(\frac{\lambda}{8m^2} \right)^n U_k^{(\pm)}(x)^{2n+1}$$

with

$$(3) \quad \begin{cases} U_k^{(\pm)}(x) = A_k^{(\pm)}(D\omega V)^{-\frac{1}{2}} \exp[\pm i\bar{k} \cdot x], \\ k \cdot x = k_0 \omega_0 - \bar{k} \cdot \bar{x}, \\ k^2 = k_0^2 - \bar{k}^2 + \omega^2 - \bar{\omega}^2 = m^2. \end{cases}$$

⁽¹⁾ P. B. BURT: *Quantum Mechanics and Nonlinear Waves* (Harwood Academic Publ., New York N. Y., 1981).

⁽²⁾ P. B. BURT: *Lett. Nuovo Cimento*, **28**, 5 (1980).

⁽³⁾ P. B. BURT: *Phys. Rev. Lett.*, **32**, 1080 (1974).

$A^{(\pm)}$ are annihilation and creation operators which satisfy commutation relations

$$(4) \quad [A_k^{(+)}, A_q^{(-)}] = \delta_{\bar{n}_k, \bar{n}_q},$$

where

$$(5) \quad \bar{n}_k = 2\pi V^{-\frac{1}{3}}(n_1 \hat{e}_1 + n_2 \hat{e}_2 + n_3 \hat{e}_3).$$

V is the volume of the system and can be absorbed by redefining the coupling constant as $\lambda' = \lambda/V$.

The series expansion for the solution is asymptotic and will be truncated. By requiring the total probability for creation of intermediate states to be unity, the truncation procedure will define the arbitrary momentum-dependent constant D which has the general form ⁽¹⁾

$$(6) \quad D = a_N \left(1 + 8^2 \frac{(\bar{q}^2 + (2n + 1)^2)}{\gamma^2 (2n + 1)^2} \right) \quad \text{with} \quad \gamma = \left(\frac{\lambda'}{8m^3} \right).$$

If no perturbative contributions to meson are included the constant a_N will satisfy ⁽¹⁾

$$(7) \quad 1 = \sum_{n=0}^N \gamma^{2n} \left(\frac{(2n)!}{(2n + 1)} \right) a_N^{-2n-1} \left[\frac{1}{1 + 1/8^2 \gamma^2} \right]^{2n+1},$$

where N is the label of the last term retained in the asymptotic series. Using this prescription for finding a_N ensures that the perturbation expansions for amplitudes and cross-sections remain finite.

The propagator for this theory has been constructed ^(1,3) and in momentum space is given by

$$(8) \quad P^{\text{pers}}(q) = \sum_{n=0} \frac{(2n + 1)! (2n + 1)^{2n-2} \gamma^{2n} D^{-(2n+1)}}{(q^2 - (2n + 1)^2 m^2 + i\epsilon) (\bar{q}^2 + (2n + 1)^2 m^2)^n}$$

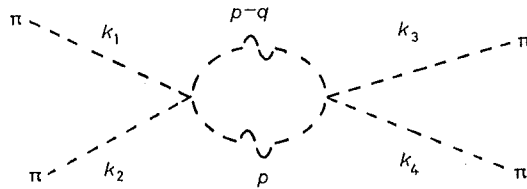


Fig. 1. - Leading-order diagram for pion-pion scattering with nonlinear propagator.

with D given in eq. (6). It is this propagator rather than the free Kline-Gordon propagator which, when used in perturbation theory, gives rise to finite-scattering amplitudes. For meson-meson s -channel scattering the first-order Feynman diagram, shown in fig. 1 gives, using persistently interacting propagators, an amplitude of

$$(9) \quad I_{l,n} = \int d^4q \sum_{n,l} \frac{C_{n,l} \{q^2 - (2n + 1)^2\}^{-1} \{(p - q)^2 - (2l + 1)^2\}^{-1}}{\{\bar{q}^2 + (2n + 1)^2\}^n \{(\bar{p} - \bar{q})^2 + (2l + 1)^2\}^l D_{(n)}^{2n+1} D_{(l)}^{2l+1}}$$

with

$$(10) \quad \begin{cases} D_{(n)} = a_N \left[1 + \frac{8^2(\bar{q}^2 + (2n + 1)^2)}{\gamma^2(2n + 1)^2} \right], \\ D_{(l)} = a_L \left[1 + \frac{8^2((\bar{q} + \bar{p})^2 + (2l + 1)^2)}{\gamma^2(2l + 1)^2} \right], \\ C_{n,l} = \gamma^{2n+2l}(2n + 1)!(2n + 1)^{2n-1}(2l + 1)!(2l + 1)^{2l-1}. \end{cases}$$

The time and angular integration can be done in the centre-of-mass frame with choice of momentum transfer equal to zero. The remaining integration is done numerically out to order $l, n = 10$ using eq. (7) to determine values of a_N and a . Using Feynman rules (*), the cross-section is

$$(11) \quad \frac{d\sigma}{d\Omega} = \frac{|M|^2 dp_1^3 dp_2^3 \delta^{(4)}(p_a + p_b - (p_1 + p_2))}{v_{rel} 2E_a E_b (2\pi)^2 4E_1 E_2},$$

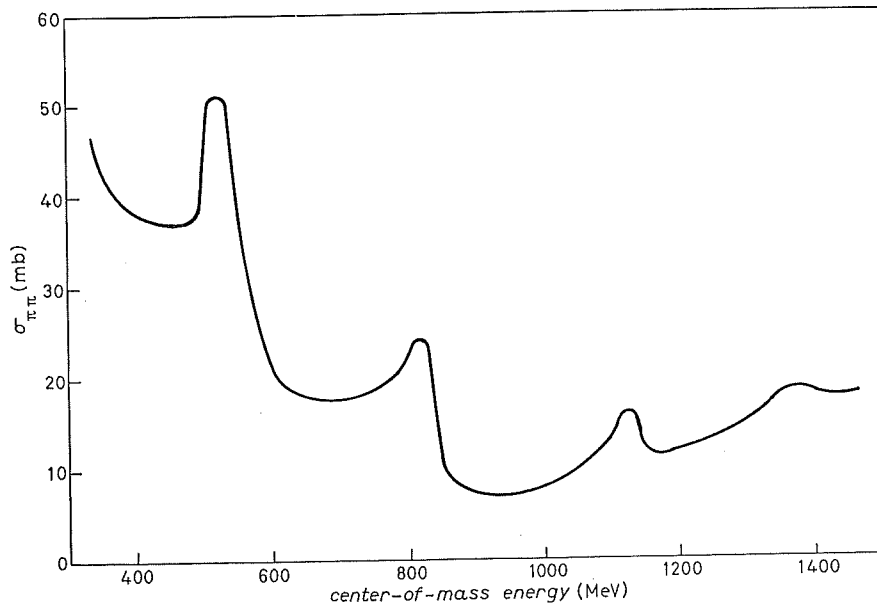


Fig. 2. - Cross-section for pion-pion scattering in millibarns for center-of-mass energies between 300 MeV to 1600 MeV with parameter γ equal to 125.

where M contains $I_{n,l}$ and isospin factors. Including these in a numerical evaluation gives the cross-section shown in fig. 2.

The general shape of the cross-section and the location of peaks is independent of the choice of coupling constant contained in the parameter γ , however the height of

(*) J. D. BJORKEN and S. D. DRELL: *Relativistic Quantum Mechanics* (McGraw-Hill, New York, N. Y., 1953).

the peaks is affected by the value of γ . Figure 2 shows the cross-section in millibarns for a choice of γ equal to 125, peak values for other choices of γ are given in table I. Below γ equal to 45 and above γ equal to 125 the values of the cross-section are unrealistically large. The effects of other channels, terms in the asymptotic series for l, n greater than 10 and perturbative contributions have not been included in this calculation.

TABLE I. - Values of the peaks in the nonperturbative contribution to pion-pion scattering cross-section in millibarns for several values of the parameter γ .

	$\gamma = 45$	$\gamma = 75$	$\gamma = 115$	$\gamma = 140$
550 MeV	$0.3660 \cdot 10^3$	$0.3874 \cdot 10^2$	$0.4778 \cdot 10^2$	$0.5081 \cdot 10^2$
820 MeV	$0.4448 \cdot 10^3$	$0.1384 \cdot 10^2$	$0.1741 \cdot 10^2$	$0.1857 \cdot 10^2$
1090 MeV	$0.1707 \cdot 10^3$	$0.8450 \cdot 10^1$	$0.1077 \cdot 10^2$	$0.1151 \cdot 10^2$
1360 MeV	$0.2966 \cdot 10^3$	$0.4860 \cdot 10^1$	$0.7342 \cdot 10^1$	$0.6807 \cdot 10^1$
1640 MeV	$0.2279 \cdot 10^2$	$0.5794 \cdot 10^1$	$0.5101 \cdot 10^1$	$0.8414 \cdot 10^1$

Experimental pion-pion cross-sections are most often obtained by extrapolation from $\mathcal{N}\pi \rightarrow \mathcal{N}\pi\pi$ processes. Although results are model-dependent extrapolations and not in exact agreement, the general features show enhancement of the cross-section, decreasing in height (⁶), at roughly 500 MeV (⁶), 800 MeV (⁷), 1300 MeV and 1600 MeV (⁸). As evidenced by results shown in fig. 2, this is in good agreement with the present calculation.

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