Wind chime physics

Kyle Forinash
Natural Science Department, Indiana University Southeast, New Albany, Indiana 47150
John P. Richie, Jr.
American Health Foundation, Valhalla, New York
Tim Jones[a)]
Natural Science Department, Indiana University Southeast, New Albany, Indiana 47150

(Received 4 December 1987; accepted for publication 12 February 1989)

The construction of a set of wind chimes tuned to specific pitches is described. A method for the determination of Young's modulus based on the measured frequencies of vibration of the sample is employed in the construction of the chimes. Some complications dealing with the perception of musical pitch are also discussed.

The frequencies of transverse waves on a pipe or bar vibrating with both ends free are given by[1] $f_n = \pi v K m^2/8L^2$, where $v = \sqrt{Y/r}$ is the speed of sound in the medium, Y is Young's modulus, r is the density, L is the length, and $m = 3.0112, 5, 7...(2n+1)$. Here, $n = 1$ labels the fundamental and $n > 1$ is the overtone number. For a tube, the radius of gyration K equals $\frac{1}{2}\sqrt{a^2 + b^2}$ where a and b are the inner and outer radii of the tube. Using this formula it is possible to determine the length of pipe necessary to produce a desired frequency of vibration.

Sets of wind chimes were constructed from galvanized pipe with densities determined by Archimedes' principle using small samples of the pipe. A value of 7.8×10^3 kg/m^3 was found, which matches that of steel or iron.[2]

A first estimate of Young's modulus was obtained by matching the pitch of a length of pipe by ear to an electronic keyboard instrument. It was necessary to support the pipe at the location of the node[3] so that all overtones are damped except one. This method turns out to be quite accurate for lower pitches where the ear is more sensitive to changes in frequency. Care must be taken with this method because, as discussed below, the assumption that the pitch is determined by the fundamental frequency of vibration is not always true.

A more accurate value of Young's modulus was obtained using the circuit shown in Fig. 1. Voltages from the microphone were digitized using an analog-to-digital conversion board designed by Priest and Snider[4] and a machine-language program was written to determine frequency. To ensure that only one particular overtone was measured, an oscilloscope was connected to the circuit and the support at nodes was adjusted to give pure sine waveforms. An average value for Young's modulus of 196×10^9 N/m^2, calculated from the measured frequencies shown in Table I, matches the value of Young's modulus for iron.[2] The method described here for determining Young's modulus is the inverse of that reported by Tyagy and Lord,[5] using measured frequencies as output rather than input, and works well for larger samples in the shape of a bar or tube.

The chimes we constructed are described in Table I. The perceived musical pitches listed in the table were determined by having professional musicians match the sound of a finished chime (freely hung from the node of the fundamental) to notes on a keyboard instrument. (This is somewhat difficult because pipes and keyboard instruments both have their own overtones. We had two music faculty members and a music therapist make comparisons to pianos and a local musician make comparisons to a synthesizer.) According to these measurements, the perceived pitches of the pipes listed in the table do not correspond to the fundamental frequency of vibration as measured by the computer. For the first pipe listed, this is the result of a phenomenon in the perception of complex tones known as the missing fundamental.[6,7] In this effect, it is possible to remove the fundamental of a harmonic sequence of tones (whole number multiples of the fundamental) and still have that pitch be perceived.[8,9] For example, if frequencies of 800, 1000, and 1200 Hz are combined, most listeners will match the resulting sound to a pure frequency of 200 Hz. If the overtones are not harmonic (such as for a vibrating pipe), it can be the case that the perceived pitch is not directly related to the fundamental. For example, the overtones of musical chimes used by orchestras are tuned so that the third, fourth, and fifth overtones ($n = 4,5,6$ vibrational modes) combine to give a perceived pitch (called the strike tone) that is not the fundamental frequency or any overtone.[1,10] The first pipe listed in Table I has a perceived pitch corresponding to a missing fundamental based on a combination of the fourth, fifth, and sixth vibrational modes, as predicted by this effect.

At least two other factors enter into the determination of the perceived pitch. The human ear has a dominance range from approximately 500 to 2000 Hz and overtones with frequencies in this range have the largest influence on the perceived pitch.[11] The relative amplitudes of the overtones also play a role in the determination of perceived pitch. For the other pipes listed in Table I, our experts determined a perceived pitch corresponding to the first overtone (the $n = 2$ vibrational mode). The frequencies of the fourth, fifth, and sixth vibrational modes and the fundamental fre-

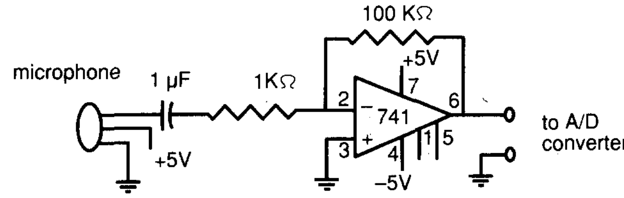

Fig. 1. Schematic diagram of microphone and amplifying circuit.

Table I. Values of measured frequencies for several lengths of galvanized iron pipe with i.d. 3.5 cm and o.d. 3.8 cm. Notice that the perceived pitch does not correspond to the fundamental frequency (see text for explanation).

Pipe length (m)	Fundamental node (m, from top)	Fundamental (Hz) f_1	Overtones (Hz) f_2	f_3	Perceived musical note
1.71	0.385	72	213	449	F4
1.33	0.298	140	378	714	F#4
0.902	0.202	287	764	1588	G5
0.849	0.190	320	857		A5
0.803	0.180	360	952		B6
0.779	0.174	381	1009		C6
0.736	0.165	438	1136		D6

quency lie outside the dominance range of human hearing for these shorter pipes and it is our belief that this results in a perceived pitch that corresponds to f_2, rather than a combination of overtones. The $n = 3$ mode is approximately an octave higher than f_2 so that these two frequencies correspond to the same musical note.

We found a collection of pipes with lengths 90.2, 84.9, 77.9, and 73.6 cm corresponding (approximately) to perceived musical notes G5, A5, C6, and D6 to produce a pleasant, restful sound for a set of wind chimes. Many other pleasing combinations are possible.

[a] Currently at Purdue University.

[1] T. D. Rossing, "Acoustics of percussion instruments-Part I," Phys. Teach. **14**, 546–556 (1976).

[2] *American Institute of Physics Handbook* (McGraw-Hill, New York, 1982), pp. 2–20 and 2–68.

[3] H. F. Olson, *Music Physics and Engineering* (Dover, New York, 1967).

[4] J. Priest and J. Snider, "Undergraduate computer-interfacing projects," Phys. Teach. **25**, 303–313 (1987).

[5] S. Tyagy and A. E. Lord, "Simple and inexpensive apparatus for Young's modulus measurement," Am. J. Phys. **48**, 205 (1980).

[6] T. M. Kalotas and A. R. Lee, "Fun with missing fundamentals," Am. J. Phys. **55**, 184 (1987).

[7] C. W. Leming, "Simple demonstration of "missing fundamental" in harmonically related sounds," Am. J. Phys. **52**, 470 (1984).

[8] B. C. Moore, *Introduction to the Psychology of Hearing* (Academic, New York, 1982).

[9] T. D. Rossing, "Resource Letter MA-2: Musical acoustics," Am. J. Phys. **55**, 589–601 (1987).

[10] T. D. Rossing, *The Science of Sound* (Addison-Wesley, Reading, MA, 1982).

[11] R. A. Rasch and R. Plomp, "The perception of musical tones," in *The Psychology of Music*, edited by D. Deutsch (Academic, New York, 1982).

Generalization of the Hagen–Poiseuille velocity profile to non-Newtonian fluids and measurement of their viscosity

Byung Chan Eu[a)]

Department of Chemistry, McGill University, 801 Sherbrooke Street West, Montreal, Quebec H3A 2K6, Canada

(Received 2 June 1988; accepted for publication 7 February 1989)

By using the constitutive equation for the stress tensor that yields a viscosity formula similar to the Eyring formula for non-Newtonian fluids and solving the hydrodynamic equation for a tube flow of such fluids, a generalization is obtained of the Hagen–Poiseuille formula for velocity profile in tube flow under pressure. The volume flow rate is calculated with the velocity profile, and a method of measuring zero shear rate viscosity is suggested. As the pressure difference increases, the calculated velocity profiles become increasingly flatter in the midportion of the tube than that predicted by the Hagen–Poiseuille formula, which remains parabolic for all values of the pressure difference.

It is well known[1,2] that when a viscous fluid flows laminarly in a circular tube, the velocity profile of the flow is parabolic and the volume rate of flow is proportional to $R^4 \Delta p/\eta_0 L$, where R is the radius of the tube, Δp is the pressure difference between the head and the end of the tube, L is the length of the tube, and η_0 is the viscosity of the fluid. The velocity profile is known as the Hagen–Poiseuille velocity profile and provides a theory of measurement for viscosity.[1] Many modern physical chemistry textbooks[1a] discuss the theory, and in fact the Ostwald viscometer is based on the Hagen–Poiseuille velocity profile. The theory is based on the assumptions that the fluid is incompressible, Newtonian, non-heat-conducting, of uniform temperature, and the shear rate is not too large so that the flow is laminar. A fluid is called Newtonian if the viscosity is independent of the shear rate. The Navier–Stokes equation in conventional hydrodynamics[2] is based on the implicit assumption of Newtonian viscosity for the fluid, although this point is rarely mentioned in fluid dynamics textbooks. However, many fluids of practical interest such as polymer solutions, polymer melts, and colloidal suspensions are not Newtonian since their viscosity is in general dependent[3] on the shear rate. Such fluids are called non-Newtonian. For such fluids, the assumption of Newtonian viscosity made for the Hagen–Poiseuille velocity profile becomes invalid, and the velocity profiles are nonparabolic as in the case of, for example, the Bingham fluid[4] that models paints and plastics. A nonparabolic flow profile is also shown[5] in the