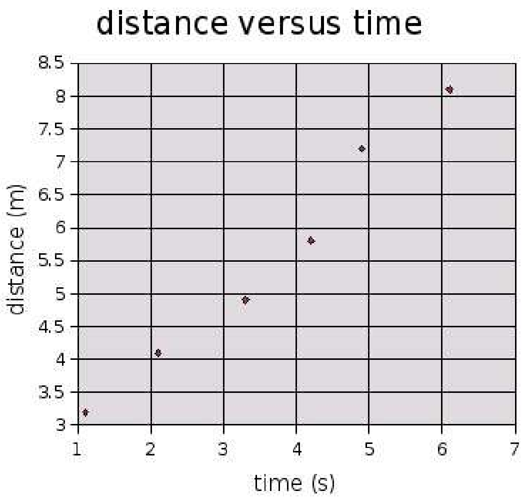
NOTES2: **Kinematics (describing how things move)**

- These are supplementary notes only, they do not take the place of reading the text book.
- Like learning to ride a bicycle, you can only learn physics by practicing. There are worked examples in the book, homework problems, problems worked in class, and problems worked in the student study guide to help you practice.

Key concepts:

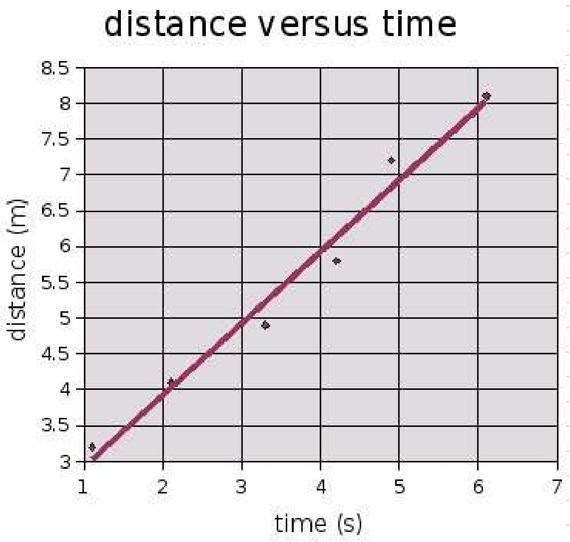
Suppose you have a toy car that goes across the floor and, using a meter stick and a stop watch, you get the following data where column B has the locations of the car at the times shown in column A:

	A	B
1	1.1	3.2
2	2.1	4.1
3	3.3	4.9
4	4.2	5.8
5	4.9	7.2
6	6.1	8.1



What does this list of numbers tell us? Not much! So let's graph the data (right):

From the graph we can see the data forms a straight line, more or less. So let's do a linear regression (use some math or a computer to find the best fitting straight line):



What do we gain by having the line? We can write an equation for a line. This allows us to summarize the data in a single formula.

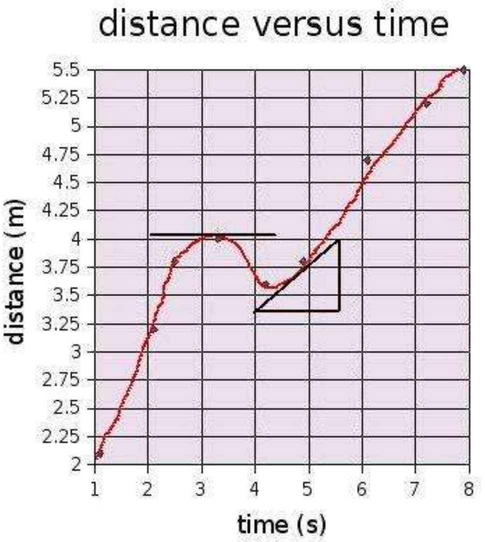
The generic equation for a straight line is $y = mx+b$ where y represents numbers plotted on the vertical axis (the x distance traveled in this case), x is what is plotted on the vertical axis (time in this case), b is the vertical – intercept (where the line crosses the vertical axis; 3m in this case) and m is the slope of the graph.

1. Normally we have the computer calculate the slope for us (because it is easier)

but it is instructive to note that we can get the slope from the rise over the run. In the time between 1s and 6s the line rises from 3m to 8m. The slope is therefore (8m - 3m)/(6s - 1s) = 1m/s.

2. So the equation (y = mx + b) becomes for this particular data x = 1m/s t + 3m.
3. Notice that the slope of this graph is in the units of m/s which is a speed. This is always true, the slope of a distance versus time graph is speed.
4. You will notice that the slope can also be found by taking the rise for a 1s interval (5m - 4m)/(3s - 2s) = 1m/s. In fact the slope is the same for any interval along the line. For example (7m - 5m)/(5s - 3s) = 1 m/s.
5. Now we have much more information than we had with just the numbers in the table. If we want to know where the car is at 2.5s we can plug that into the equation x = 1m/s t + 3m and calculate the location. This is called interpolation. We can also extrapolate to times after 7s to find out where the car would be.
6. Notice that if the data had resulted in a line tilting downward the slope would be negative (a negative rise over a positive time change). This would be the case if the toy car were traveling towards the x = 0 location instead of away from it. In other words, a negative slope (negative velocity) represents an object moving in the -x direction.

What if the data had not resulted in a straight line?

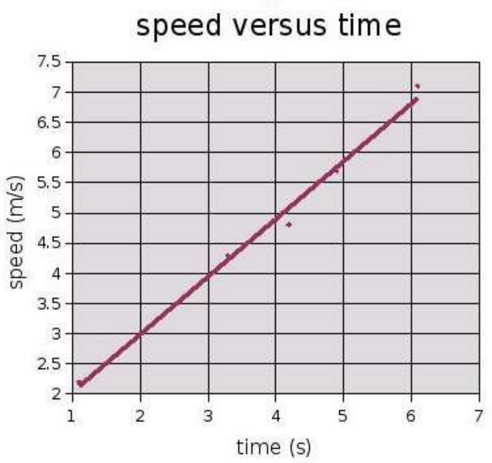


1. Here we see the slope changes. On top of the 'hump' the slope is zero (the tangent to the curve is horizontal). So for this instant the car stops going forward and starts moving backward. The slope is again zero right after 4s when the car stops going backwards and starts going forward.
2. For the data point at 5s the slope of the tangent line is (4m – 3.3m)/(5.5m – 4m) = 0.47m/s. This is defined to be the instantaneous speed at that point (5s).
3. The above discussion allows us to make two definitions of speed, both of which are based on the slope. Average speed is $v_{ave}=\frac{x_f-x_i}{t_f-t_i}=\frac{\Delta x}{\Delta t}$ where the f and i refer to the final and initial locations and times (regardless of what happens in between). So average speed is the average slope between two points.
4. Instantaneous speed, the speed at a single point in time, is defined to be $v=\lim_{\Delta t\rightarrow 0}\frac{\Delta x}{\Delta t}=\frac{dx}{dt}$. The instantaneous speed is thus the same thing as the slope of the tangent line at that point. In calculus the tangent at a point is known as the derivative. So instantaneous speed is the derivative of location with respect to time. (You may be worried about the denominator going to zero

here. Don't! As the denominator gets smaller Δx also gets smaller so the ratio stays fixed.)

5. Note that if the initial position and initial time is zero the average velocity equation is $v_{ave} = x/t$ or, rearranging $x = vt$ (rate times time gives distance). Caution! This equation is for average speed only. If there is acceleration you may not be able to use it since you would have to know the starting an ending speeds.

What about other kinds of graphs? Suppose we did a different experiment and plotted speed versus time and got the following graph:



1. First note that, although the line is straight again the speed is changing over time unlike in the first graph where a straight line meant a constant speed.
2. Because it is a straight line we can again make an equation of this data using $y = mx + b$. Only this time speed is on the vertical axis. The vertical intercept will be an initial speed instead of the initial position.
3. Slope here will be in units of m/s^2 because the slope is the change in speed over the change in time. This tells us that the slope of all speed versus time graphs is acceleration. For this data it is approximately $1 m/s^2$.
4. We can again define average acceleration as $a_{ave}=\frac{\Delta v}{\Delta t}$ and instantaneous acceleration as $a=\lim_{\Delta t\to 0}\frac{\Delta v}{\Delta t}=\frac{dv}{dt}$. So instantaneous acceleration is the slope of a speed versus time graph or the derivative of speed with respect to time.
5. Our equation for this data is thus $v = 1 m/s^2\, t + 2 m/s$.

So far we know that if we see a straight line on a distance versus time plot the equation is $x = v_o t + x_o$ where v_o is a constant speed and x_o is the initial position. If we see a straight line on a speed versus time plot the equation is $v = at + v_o$ where a is the constant acceleration and v_o is the initial velocity.

What would a constant acceleration look like on a distance versus time graph? Well it is not going to be straight.

1. Here is one way to figure out what the distance equation for constant acceleration would look like. Use $v_{ave} = x/t$ and to use the definition of an average as $v_{ave} = (v – v_o)/2$. Putting the second into the first we have $x = ½ (v_o+at + v_o)t$ or $x = x_o + v_o t + ½ at^2$.
2. A second way is to take the equation for a straight line on the speed versus time graph and recall that speed is defined to be the derivative of position. We can take the integral (anti-derivative) of that equation to get an equation with x

and t. Here is what we get: $x = x_o + v_o t + ½ at^2$.

3. We can get one more equation for accelerated motion if we use $v = at + v_o$ to get $t = (v - v_o)/t$ and insert this and the average speed ($v_{ave} = (v – v_o)/2$) into $v_{ave} = x/t$. This give us, after some rearranging, $v^2 – v_o^2 = 2ax$.

So we have four equations which we can use to describe the motion of an object:

1. $x = v_o t + x_o$ (where v_o is the average speed: Caution! If there is acceleration you have to use the average speed!).
2. $v = at + v_o$ (if there is constant acceleration).
3. $x = x_o + v_o t + ½ at^2$ (if there is constant acceleration).
4. $v^2 – v_o^2 = 2ax$ (if there is constant acceleration).

All of the above applies to motion in the x direction (linear motion). We can also define distances, speeds, and accelerations in the y and z directions. In other words the same equations apply for the components of the position vector ($\mathbf{r} = x\mathbf{i} + yj + z\mathbf{k}$), velocity vector ($\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}$) and acceleration vector ($\mathbf{a} = a_x i + a_y j + a_z\mathbf{k}$). This is why we have been using the symbol v for speed; v_x is the speed in the x direction, v_y is the speed in the y direction, etc.

5. $y = v_{yo} t + y_o$ (where v_{yo} is the average speed of the y component of velocity: Caution! If there is acceleration you have to use the average velocity!).
6. $v_y = a_y t + v_{yo}$ (if there is constant acceleration in the y direction).
7. $y = y_o + v_{yo} t + ½ a_y t^2$ (if there is constant acceleration in the y direction).
8. $v_y^2 – v_{yo}^2 = 2a_y y$ (if there is constant acceleration in the y direction).

9. A similar set of four equations for z and z components of velocity and acceleration.

<u>Caution!</u> Notice that you **NEVER EVER, EVER** see x components in the y or z equations and vice versus. The components are COMPLETELY independent from each other. Don't make the mistake of trying to use an x component of velocity in a y equation or a z acceleration in a y equation, etc.

The only other kinematics equations we will need this semester are for circular motion (which are very similar but in polar coordinates).

Applications and examples done in class, on quizzes, etc:

1. Lots of problems on linear motion (only in the x direction).
2. We will use the same equations for projectile and three dimensional motion.