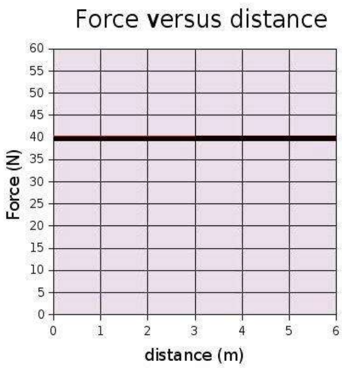
NOTES 5: **Conservation of energy.**

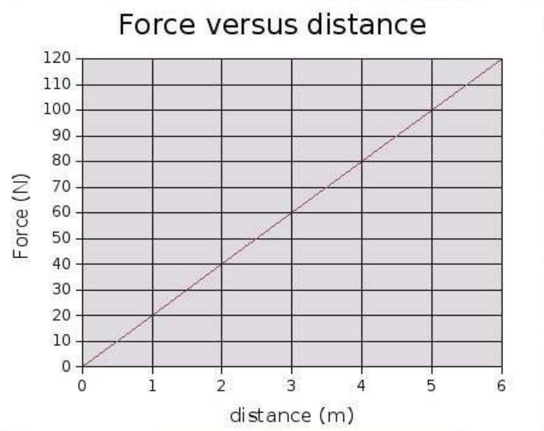
- These are supplementary notes only, they do not take the place of reading the text book.
- Like learning to ride a bicycle, you can only learn physics by practicing. There are worked examples in the book, homework problems, problems worked in class, and problems worked in the student study guide to help you practice.

Key concepts:

1. Work is defined to be the scalar product of force times the distance traveled: W = **F** · **d** = F d cos θ where **d** is the displacement vector (not the distance traveled in all cases).
2. A Newton meter is a Joule; the units of energy and work are Joules.
3. Notice here that, unlike Newton's second law where **F** represents the net force (sum of all forces in each direction) in this definition for work the **F** is a single force (you get to pick which one).
4. Notice also that if the force you pick is perpendicular to the distance traveled (θ = 90°) that particular force does no work (some other force might still do work but not the one perpendicular to the motion).
5. There is a problem with the above definition. What do we do if the force varies over the distance traveled? First lets make a plot of force versus distance. For a constant force of 40N which moves an object from 0m to 6m we have:



   Notice that according to the definition, the work is the area under the line between the beginning and ending points: 40N x 6m = 240J. In calculus the area under a line on a graph is the definition of the integral. So a better definition of work is: $W=\int_{x_i}^{x_f}\mathbf{F}\cdot\mathbf{ds}$ where x_i and x_f are the beginning and ending points (0m and 6m in the example above) and **ds** is a very tiny displacement.
6. Now lets look at a force which varies as you move. An example is a Hooke's law force for a spring which is proportional to the distance the spring is stretched or compressed (the amount of force needed to stretch the spring gets larger the more the spring stretches). We have F = -kx where k is the spring constant (how stiff the spring is) in kg/s^2. The minus sign indicates that the spring pushes or pulls back on whatever is compressing or stretching it (opposite the direction of the force that causes the compression or stretch). Let's suppose the spring constant is k = $20kg/s^2$ and we stretch the spring from 0m to 6m (so the stretch is in the same direction as the force and θ = 0). One way to find the work is ways to calculate the integral: $W=\int_{x_i}^{x_f}-kx\,ds=\frac{-1}{2}k(x_f^2-x_i^2)=\frac{-1}{2}20\,kg/s^2((6\,m)^2-0)=-360\,J$ The minus sign tells us it is work done on the spring. The work done by the spring is +360J. A second way to find the work done is to find the area under the curve (which for a triangle is ½ hb where h is the height and b the base of the triangle):



Notice that since the area of a triangle is half base times the height you get the same result. 360J.

7. If the force is constant then the definition with the integral reduces to the first definition since the constant force would come outside the integral and $\int_{x_i}^{x_f} ds = \text{total displacement} = \mathbf{d}$.
8. What do we do if we have the potential energy and want to find the force? We have to un-do the integral. The inverse operation of integration is the derivative. So we have $F_x = -\frac{\partial U}{\partial x}$ where U is the potential energy. For example the energy stored in a spring is U = ½ mv^2. So $$F_x = -\frac{\partial(\frac{1}{2}kx^2)}{\partial x} = -kx\ .$$
9. To find the force in the y direction we take a y derivative: $F_y = -\frac{\partial U}{\partial y}$ (this would be zero for the spring). The derivative signs here looks a little different than usual (it is not dU/dx). Why? It is to remind us that the potential, U, could be a function of x, y and/or z. When we want the force in the x direction we take only the x derivative, pretending that y and z are constant. This kind of derivative is called a partial derivative.
10. In the case of a turning force (a torque) the definition of work (in Joules) is: $\int_{\theta_i}^{\theta_f} \tau \times \mathbf{d}\theta$ where **d**θ is a tiny angular displacement along the path between the initial and final angles.

It is possible to store up energy by doing work on an object.

1. If you apply a force over a distance (do work) and then let go of the object and it is moving we say it has kinetic energy (KE = ½ mv^2).
2. If you lift an object against the force of gravity (mg) you store up gravitational potential energy. Near the surface of the earth this would be PE = mgh where h is how high you lift the object from its starting point and g is the acceleration of gravity (no minus sign).
3. If you compress or stretch a spring you store energy in the spring: SPE = ½ k x^2 as we showed above.
4. If a friction force, f, operates then some work is turned into heat energy (for example rubbing your hands together makes them warmer). This energy is now in a form that in general cannot be recovered for useful purposes. The amount of energy converted into heat in this case is W = f d where d is the distance over which the friction force acts.
5. Temperature is defined to be a measure of the random thermal motion of the

individual atoms and molecules making up a substance. $\langle \frac{1}{2} m v^2 \rangle = \frac{3}{2} k_B T$ where T is the temperature in Kelvin, k_B = 1.38 x 10^{-23} J/K is Boltzmann's constant and the brackets (<>) mean the average kinetic energy.

6. Heat is defined to be the amount of energy transfered due to a temperature difference (difference in the average kinetic energy).
7. If you apply a torque to something to make it rotate you do work on it and the stored kinetic energy is equal to $KE_{rot} = ½ I \omega^2$ where I is the moment of inertia and ω is the angular velocity.
8. Notice that the units for all of the above (including heat) are Joules, J = Nm = kgm^2/s^2 except temperature which is in Kelvin. Notice also that work and energy are scalars (NOT vectors).

Using the above definitions we can write the first of a dozen or so conservation laws found in physics. All of physics can be summed up in these conservation laws.

conservation of energy:

Energy at the beginning of a process = energy at the end plus (or minus) any input (output) energy (or work or heat).

The way to solve all the problems in this section is to:.

1. Pick a point in the process to call the 'before' picture and add up all the energy (kinetic, work, potential, rotational, etc.).
2. Pick a point in the process to call the 'after' picture and add up all the energy (kinetic, work, potential, rotational, etc.).
3. Set the two sums equal to each other.
4. The 'before' and 'after' pictures can be any two times (so 'after' could really be half way through or two thirds of the way through or ....).

Power is the rate at which work is done or work done divided by the time taken: $P = \frac{\partial W}{\partial t}$. The units are J/s = Watt (W). 1 hp = 746W. Since W = **F** · **d** and the derivative of time is velocity another definition of power is $P = \mathbf{F} \cdot \mathbf{v}$.

Applications and examples done in class, on quizzes, etc:

1. First we will work problems using the various definitions of energy and work.
2. Then we will work conservation of energy problems of increasing complexity (more terms in the 'before' and 'after' pictures).
3. Problems relating force to potential, U, will be useful later so we will do some of those too.