NOTES4: **Dynamics: Why things move.**

- These are supplementary notes only, they do not take the place of reading the text book.
- Like learning to ride a bicycle, you can only learn physics by practicing. There are worked examples in the book, homework problems, problems worked in class, and problems worked in the student study guide to help you practice.

Key concepts:

There are four fundamental forces:
1. Gravity.
2. Electromagnetism.
3. Strong nuclear force.
4. Weak nuclear force.

All other forces (friction, Hooke's law, etc.) can be explained in terms of these four fundamental forces. Forces are vectors.

The equation for gravity looks very similar to the equation for the static electric force (part of the electromagnetic force).
1. The gravitational force attracts any two masses towards each other: $F = -\frac{G m_1 m_2}{r^2}$ Where m_1 and m_2 are the two masses, r is the distance between their centers and G = 6.63x10^{-11} Nm^2/kg^2 is a universal constant. Note that for a small mass m near the surface of the earth (use the earth's radius for r and the earths mass for m_2) you get F = m (9.8 m/s^2).
2. The electromagnetic force attracts or repels any two charges toward each other: $F = \frac{k q_1 q_2}{r^2}$ Where q_1 and q_2 are the two charges, r is the distance between their centers and k = 8.98x10^9 Nm^2/C^2 is a universal constant. Since q can be positive or negative (unlike mass which is always positive), the electrical force can be repulsive)a negative sign makes the force point in the opposite direction. Since k is so much bigger than G we can easily demonstrate the electrical force but the gravitational force only shows up between very large objects (for example us and the earth).
3. Another force we will encounter is Hooke's law for springs. Most springs pull or push with a force proportional to the distance the spring is stretched or compressed: F = -kx where k is the spring constant and indicates how stiff the spring is. The units of k are N/m.
4. Friction is the force between and object and a surfaces that causes a resistance to moving the object: F = μ N where N is the normal force (the force perpendicular to the surface that pushes the two objects together). The coefficient of friction μ indicates how sticky the surface is and has no units. There are actually two coefficients, kinetic and static, depending on whether the object is sliding over the surface or not. The coefficient of static friction for a given surface is larger than the coefficient of kinetic friction so you will get more friction force from your tires if you do not allow them to slide. Both coefficients are very rarely larger than 1.

There are three laws governing forces (Newton's laws):

1. Newton's first law: Objects continue in straight line motion with constant velocity unless acted on by a force.
2. Newton's second law: **F** = m**a**. Forces cause acceleration (<u>not</u> velocity). Here **F** represents the net force (sum of all forces acting in all directions) and is a vector.

3. Newton's third law: If an object applies a force on a second object, the second object applies a force of equal magnitude on the first object but in opposite direction. Notes: a) Each of the two forces always act on a different objects. b) The effect of the forces on the two objects may be different. For example if a car hits a bug the force is the same on bug and car but that force squashes the bug but does almost nothing to the car.

There are two ways to multiply vectors:

1. The scalar product multiplies two vectors and gets a scalar (just a number) as a result. $\mathbf{A} \cdot \mathbf{B} = AB \cos\theta = A_xB_x + A_yB_y + A_zB_z$. All the information about direction of the vectors is lost.
2. We will use this in the definition of work in the next chapter: $W = \mathbf{F} \cdot \mathbf{ds} = F\, ds \cos\theta$.
3. The vector or cross product multiplies two vectors to get a new vector as a result. $\mathbf{A} \times \mathbf{B} = (A_yB_z - B_yA_z)\mathbf{i} + (A_zB_x - A_zB_x)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}$.
4. The direction of this new vector will be perpendicular to the plane of the two initial vectors **A** and **B** using the right hand rule: Point your thumb in the direction of the first vector and your fingers in the direction of the second, your palm will face the direction of the new vector. Notice that the order of the vectors does not matter for the scalar product but does matter for the vector product (where reversing the order gives a minus sign).
5. An example of where we use the cross product is the torque: $\tau = \mathbf{r} \times \mathbf{F}$. Torque can be thought of as a force that causes something to turn (a turning force). The vector **r** is called the lever arm and is the vector that starts at the pivot point (around which the object will rotate) and goes to the location where the force acts. The torque vector points along the axis of rotation, using the right hand rule. You will notice that turning forces which act in opposite directions (clockwise versus counterclockwise for example) will end up with opposite signs because of the right hand rule (so opposite torques tend to cancel each other).
6. Notice that work and torque both multiply a force times a distance and are therefore in the same units (Newtons times meters) but they represent different physical objects. We use the scalar and cross products to create different mathematical objects which represent different objects in the physical world.

Rotation:

1. Mass can be thought of as the resistance to being accelerated and is sometimes called inertia. The bigger the mass the more resistance to being accelerated and object has. The resistance to being rotated is called moment of inertia, I, and depends not only on the mass but also on the where the mass is located relative to the pivot point. A wheel with all the mass out on the rim has a larger moment of inertia (is harder to accelerate) than one of the same radius with most of the mass near the center. The units of moment of inertial are kgm^2. There are various formulas for calculating the moment of inertia but we won't worry about those.
2. Newton's laws also can be applied to objects moving in a circle. The second law ($\mathbf{F} = m\mathbf{a}$) then looks like $\tau = I\alpha$ where α is the angular acceleration (defined in NOTES3) and I is the moment of inertial. Torque can be thought of as a force that causes turning to occur, moment of inertia as the resistance to turning and angular acceleration as the rotational acceleration due to the turning force.

The two conditions for static equilibrium are that:

- The sum of all forces add to be zero (there is no acceleration).
- The sum of all torques add to zero (there is no angular acceleration).

If it is assumed that the initial velocity is zero then an object in static equilibrium remains at rest.

Applications and examples done in class, on quizzes, etc.

We will do lots of problems involving Newton's laws.
We will use the scalar and cross product a lot (particularly in P222) and do many problems using them.
We will only mention torque and static equilibrium briefly.

Supplementary material (we won't cover this in class but you will see an example in the lab).

How to calculate moment of inertia.

Suppose we have a small mass moving around in a circle and it has a tangent acceleration a_T (it is getting faster as it goes around). We can write $F_T = ma_T$ where F_T is the tangent force that causes it to speed up. If we multiply both sides by the radius of the circle we have $r F_T = ma_T r$. Note that on the left we have a torque (since r and F are perpendicular). On the right using $a_T = \alpha r$ we have $\tau = mr^2\alpha$**.** This would look similar to F = ma if we defined $I = mr^2$ to be something like inertia, called moment of inertia. This gives us $\tau = I\alpha$, the angular equivalent to F = ma.

As shown above, $I = mr^2$ is the moment of inertia for a small mass traveling in a circle. The general formula for moment of inertia is: $I = \int r^2 dm$
What this means physically is to imagine chopping the object into tiny pieces each of mass dm (think Δm) at a distance r from the pivot point. Add each piece times the distance to that piece squared.

Here is and example. Imagine a uniform disk of total radius R and mass per unit area (m/A_{tot}) of λ. Chop the disk into rings of radius r where each ring has a width of dr and circumference of $2\pi r$ (r is a variable here- each ring has a different r). The mass of each ring will be its area ($2\pi r dr$) times λ or $2\pi r\lambda dr$. To find the moment of inertia we need to add the mass of all the rings together, each times r^2. So: $I = \int_0^R r^2 2\pi r \lambda dr = 2\pi\lambda \int_0^R r^3 dr = \frac{\pi\lambda R^4}{2}$ If we now use the fact that the total mass of the disk is $m = \pi\lambda R^2$ we have $I = \frac{1}{2} mR^2$ for the moment of inertia of the disk.

Some other moments of inertia for rings, spheres, etc. are given in your book.