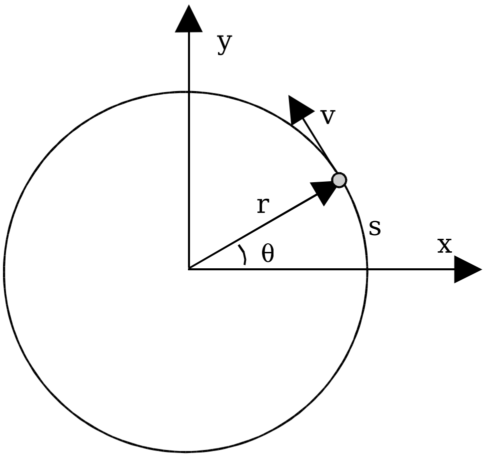
NOTES3: **Kinematics for circular motion**

- These are supplementary notes only, they do not take the place of reading the text book.
- Like learning to ride a bicycle, you can only learn physics by practicing. There are worked examples in the book, homework problems, problems worked in class, and problems worked in the student study guide to help you practice.

Key concepts:

If an object is moving in a circle it is often easier to use polar coordinates instead of x, y, z components.



Here a small object (the gray circle) travels in a circular path with speed v. We can locate the object with the usual x and y positions or we can locate the object with the the polar coordinates r and θ. The distance along the edge of the circle from the x axis to object is s.

θ must be measured in radians!

1. The relationship between s, the distance along the curve, θ (measured in radians!) and r (the radius) is: $s = r\theta$.
2. We can define an angular speed (measured in radians per second) just like we did for linear speed: $\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$.
3. The relationship between linear speed v, ω (measured in radians per second!) and r (the radius) is: $v = r\omega$.
4. <u>If</u> the object is also speeding up (accelerating) as it travels around the circle we can define an angular acceleration (measured in radians per second squared) just like we did for linear acceleration: $\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$
5. So for circular motion we can have four very similar kinematic equations except the variables are angular position, angular velocity and angular acceleration:
   - $\theta = \omega_{ave} t + \theta_0$.
   - $\theta = \omega_0 t + 1/2\alpha t^2$.
   - $\omega = \omega_0 + \alpha t$.
   - $\omega^2 - \omega_0^2 = 2\alpha\theta$.

Since the object is moving in a circle we know there will be a linear acceleration, a, in meters per second squared (only uniform straight line motion can have no acceleration). <u>If</u> the object is also getting faster as it goes around the circle there is a second linear acceleration along the edge of the circle. In other words we can break the total linear acceleration in two parts:

1. Centripetal (your book calls this radial) acceleration; the acceleration needed to keep the object moving in a circle at <u>constant</u> speed: $a_r = v_T^2/r$.

2. Tangential acceleration; the acceleration needed if the object is speeding up as it goes around the circle: $a_T = r\alpha$. Notice that this is related to the angular acceleration in the same way that θ and ω are related to linear position and speed.
3. If the object goes around the circle at constant speed the angular and tangent acceleration are both zero. The radial acceleration can never be zero for an object moving on a curved path.
4. If the object goes around the circle at constant speed there is still radial acceleration because the velocity is changing direction (but not magnitude). Suppose for example the object is traveling at a constant speed of 5m/s. At the instant the object crosses the x- axis it is moving upward so its velocity is 5m/s **j**. But when it crosses the y-axis it is going in the negative x direction so its velocity is -5m/s **i**. So if the time interval is, say, 1s the average acceleration is **a** = Δ**v**/Δt = (-5m/s **i** - 5m/s **j**)/1s or **a** = -5m/s^2 **i** - 5m/s^2 **j** (not zero).

Applications and examples done in class, on quizzes, etc.

Due to a shortage of time we will only do a few problems on angular kinematics. But we will need the definitions of angular quantities later for torque and angular momentum.